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Propagation of Thickness-Twist Waves in an Infinite Piezoelectric Plate

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1. Introduction

Thickness-twist waves in piezoelectric plates are widely used as the operating modes for acoustic wave resonators, filters, sensors, and other devices (Mindlin, 1965; Mindlin, 1967; Pearman, 1968; Bleustein, 1966; Bleustein, 1969; Yang et al., 2008a; Yang & Guo, 2006). Acoustic waves in these devices are driven by an alternating voltage applied to the electrodes deposited on the plates. Therefore there has been continuing research effort on the effect of electrodes (Ballato & Lukaszek, 1974; Kosinski, 2003; Wang & Shen, 2005; Yang et al., 2005). Recently, due to the need of device miniaturization, there has been growing research interest on electrode configurations. Electrodes of varying thickness have been used to adjust the vibration distribution in plates (Pao et al., 2007; Yang et al., 2007a; Wang et al., 2008). Electrode shape has also been analyzed for design optimization. In Yang et al. (2007b) it was shown that electrodes with corners cause field concentration and should be avoided in general. Optimal electrode size and shape were determined in Mindlin (1968) and Yang et al. (2008b). On the other hand, a vibrating elastic (or piezoelectric) body when put in contact with a viscous fluid changes its resonant frequencies due to the inertia and viscosity of the fluid. Equivalently, the speed of a propagating wave in a body in contact with a fluid is also affected by the fluid. This effect has been used to make various fluid sensors for measuring fluid viscosity or density (Kanazawa & Gordon, 1985; Josse et al., 1990; Reed et al., 2001; Kim et al., 1991; Vogt et al., 2004; Guo & Sun, 2008; Peng et al., 2006). More references can be found in a review article (Benes et al., 1995). For fluid sensor applications, vibration modes of a body without a normal displacement at its surface are of general interest. Thickness-shear and thickness-twist modes in a plate (Kanazawa & Gordon, 1985; Josse et al., 1990; Reed et al., 2001), torsional modes of a circular shaft (Kim et al., 1991; Vogt et al., 2004), and anti-plane surface waves (Guo & Sun, 2008; Peng et al., 2006) are modes with tangential surface displacements only and have been used for fluid sensor applications. To establish the relation between wave frequency and fluid density or viscosity, a coupled problem of fluid-structure interaction needs to be solved. This usually

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presents complicated mathematical problems. So far only limited theoretical results have been obtained.

In the devices studied in the above references, the electrodes are directly deposited on or attached to the surfaces of the plates. In device applications, unattached electrodes at some distance from the plate surfaces have been occasionally used (Detaint et al., 1989; Huang et al., 2001; Umeki et al., 2004). Unattached or separated electrodes offer new design parameters and certain advantages over electrodes deposited on the plate surfaces. Various undesirable effects of deposited (or attached) electrodes disappear when the electrodes are unattached. This includes, e.g., the residual stress in the electrodes and the plates, the frequency effect of electrode irregularity from manufacturing, and the field concentration at electrode edges. There are limited theoretical results on waves in piezoelectric plates with unattached electrodes. Plates of general material anisotropy with unattached electrodes were considered in Syngellakis and Lee (1993) and the results are rather complicated due to strong anisotropy. Numerical techniques had to be used to obtain quantitative results which were only given for attached electrodes. An approximate theoretical analysis on vibrations of a finite plate with unattached electrodes was given in Tiersten (1995). Pure thickness-shear modes (waves propagating in the plate thickness direction bouncing back and forth between the two surfaces) were studied in Yang et al. (2009).

In this Chapter, we study a simple and yet very useful case of thickness-twist waves propagating in an infinite piezoelectric plate with unattached electrodes. A theoretical analysis is performed using the exact equations of piezoelectricity. In order to make readers understand the problems under question easily, we first introduce the propagation of thickness-twist waves in an infinite piezoelectric plate with attached very thin electrodes. This part is mainly a reproduction of the work by Bleustein (1969), which can be used as a basic starting for those who want to conduct the research work on thickness-twist waves in an infinite piezoelectric plate. Some basic knowledge concerning thickness-twist waves and linear piezoelectricity will be introduced in this part. Then we analyze the propagation of thickness-twist waves in an infinite piezoelectric plate with air gaps between the plate surfaces and two electrodes. Dispersion relations of the waves are obtained and plotted. Results show that the wave frequency or speed is sensitive to the air gap thickness. This effect can be used to manipulate the behavior of the waves and has implications in acoustic wave devices. Last we study theoretically the propagation of thickness-twist waves in an infinite piezoelectric plate with unattached electrodes and viscous fluids between the plate surfaces and the electrodes. Based on the theories of linear piezoelectricity and viscous fluids, an equation that determines the dispersion relations of the waves is obtained, showing the dependence of the phase velocity on material and geometric parameters. Due to the viscosity of the fluid, the dispersion relations are complex in general, representing damped waves with attenuation. The dispersion relations obtained can reduce to the results of a few special cases with known results. The results are useful for developing and designing fluid sensors for measuring fluid viscosity or density.

2. Governing equations

2.1 Piezoelectric plate with attached electrodes

Consider the infinite piezoelectric plate shown in Fig. 1. The ceramic plate is poled in the x_3 direction determined by the right-hand rule from the x_1 and x_2 axes. The electrodes are shorted. The structure allows the following anti-plane motion (Bleustein, 1969)

$$u_1 = u_2 = 0, \quad u_3 = u(x_1, x_2, t), \quad (1)$$

$$\varphi = \varphi(x_1, x_2, t), \quad (2)$$

where u is the displacement vector and φ is the electric potential.

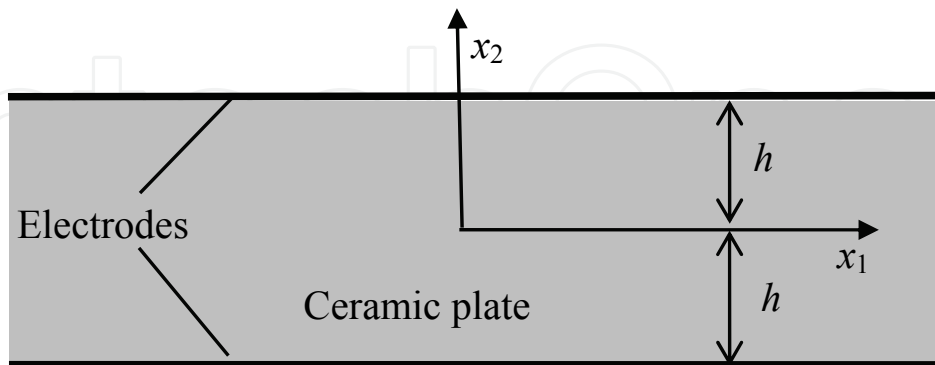


Fig. 1. A piezoelectric plate with attached electrodes

The nontrivial strain and electric field components are

$$\begin{Bmatrix} 2S_{13} \\ 2S_{23} \end{Bmatrix} = \nabla u, \quad \begin{Bmatrix} E_1 \\ E_2 \end{Bmatrix} = -\nabla \varphi, \quad (3)$$

where $\nabla = \mathbf{i}_1 \partial_1 + \mathbf{i}_2 \partial_2$ is the 2-D gradient operator. E is the electric field. The nontrivial components of the stress tensor T_{ij} and the electric displacement vector D_i are

$$\begin{Bmatrix} T_{13} \\ T_{23} \end{Bmatrix} = c_{44} \nabla u + e_{15} \nabla \varphi, \quad \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = e_{15} \nabla u - \varepsilon_{11} \nabla \varphi, \quad (4)$$

where c_{44} , e_{15} and ε_{11} are the relevant elastic, piezoelectric and dielectric constants.

A ψ can be introduced⁵ through $\varphi = \psi + eu/\varepsilon$, where $e = e_{15}$ and $\varepsilon = \varepsilon_{11}$ are the relevant piezoelectric and dielectric constants. The governing equations for u and ψ in the piezoelectric plate are

$$\left. \begin{aligned} \bar{c} \nabla^2 u &= \rho \ddot{u} \\ \nabla^2 \psi &= 0 \end{aligned} \right\}, \quad (5)$$

where $\bar{c} = c + e^2 / \varepsilon$ and $c = c_{44}$ is the relevant elastic constants. ∇^2 is the 2-D Laplacian.

2.2 Piezoelectric plate with unattached electrodes

Consider the infinite piezoelectric plate with unattached electrodes and asymmetric air gaps, as shown in Fig. 2. The piezoelectric plate is the same as that in Fig. 1. The governing equations for thickness-twist wave in such configuration are

$$\left. \begin{aligned} \bar{c} \nabla^2 u &= \rho \ddot{u} \\ \nabla^2 \psi &= 0 \end{aligned} \right\}, \quad -h \leq x_2 \leq h \quad (6)$$

$$\nabla^2 \varphi^T = 0, \quad h \leq x_2 \leq h + g^T, \quad (7)$$

$$\nabla^2 \phi^B = 0, \quad -h - g^B \leq x_2 \leq -h, \quad (8)$$

where ϕ^T and ϕ^B are the electric potentials in the top and bottom gaps, respectively.

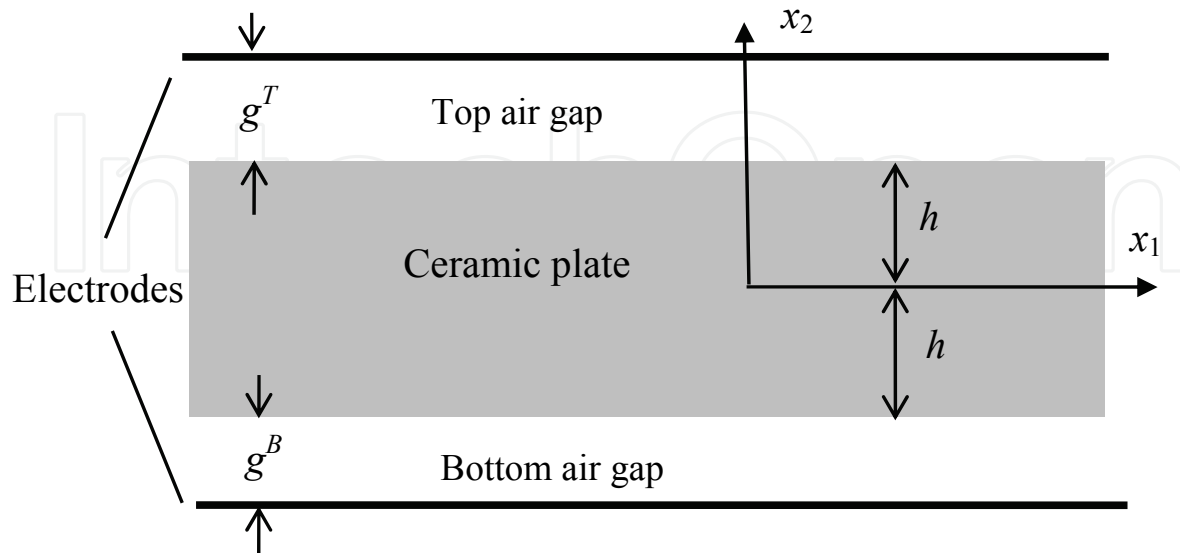


Fig. 2. A piezoelectric plate with unattached electrodes and asymmetric air gaps

2.3 Piezoelectric plate in contact with viscous fluids

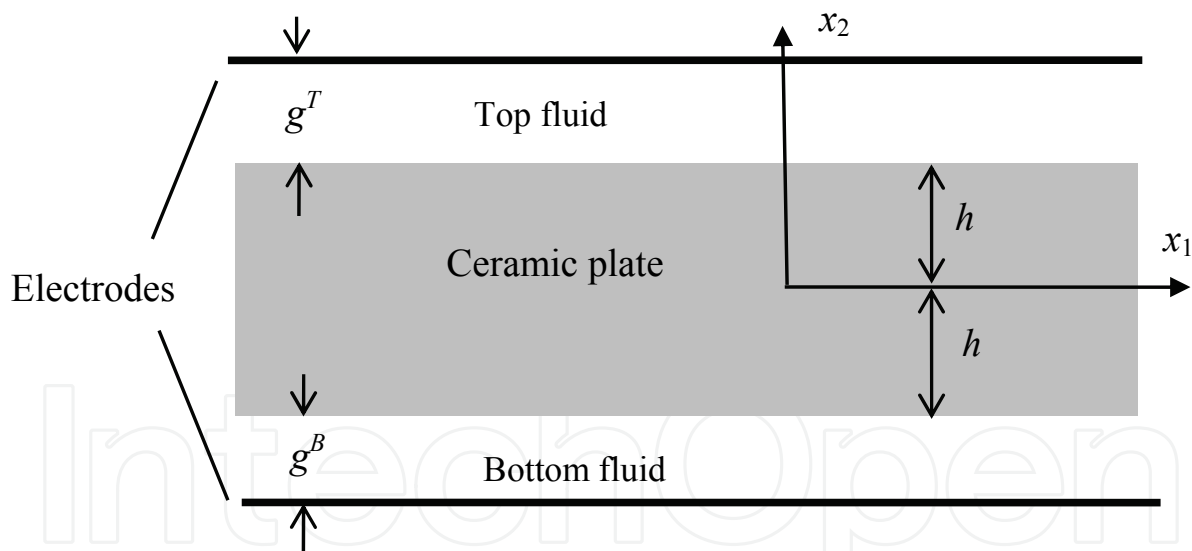


Fig. 3. A piezoelectric plate with unattached electrodes and gaps filled with fluids

Consider the infinite piezoelectric plate with unattached electrodes, as shown in Fig. 3. There are two gaps between the electrodes and the plate. In our problem the gaps are filled with two different viscous fluids. This structure includes the cases when the electrodes are on the plate surfaces or at infinity as special cases. The piezoelectric plate is the same as that in Figs. 1 and 2. The governing equations for thickness-twist wave in such configuration are

$$\begin{aligned} \bar{c} \nabla^2 u &= \rho \ddot{u} \\ \nabla^2 \psi &= 0 \end{aligned} \quad -h < x_2 < h, \quad (9)$$

$$\begin{aligned} \frac{\partial v^T}{\partial t} - \frac{\mu^T}{\rho^T} \nabla^2 v^T &= 0 & h \leq x_2 \leq h + g^T, \\ \nabla^2 \phi^T &= 0 \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{\partial v^B}{\partial t} - \frac{\mu^B}{\rho^B} \nabla^2 v^B &= 0 & -h - g^B \leq x_2 \leq -h, \\ \nabla^2 \phi^B &= 0 \end{aligned} \quad (11)$$

where v^T and v^B are the fluid velocity fields, ρ^T and ρ^B are the fluid mass densities, and μ^T and μ^B are the fluid viscosities. The superscripts "T" and "B" are for the top and bottom fluids, respectively.

3. Propagating wave solution and discussion

3.1 Piezoelectric plate with attached electrodes

The wave propagation specified by Eq. (5) must satisfy the following boundary conditions

$$\phi = 0, \quad T_{23} = 0, \quad x_2 = \pm h. \quad (12)$$

Consider the following propagating waves in the plate

$$\begin{aligned} u &= (A_1 \sin \xi_2 x_2 + A_2 \cos \xi_2 x_2) \cos(\xi_1 x_1 - \omega t), \\ \psi &= (A_3 \sinh \xi_1 x_2 + A_4 \cosh \xi_1 x_2) \cos(\xi_1 x_1 - \omega t), \end{aligned} \quad (13)$$

where A_1 through A_4 are undetermined constants, ω is the wave frequency, and ξ_1 and ξ_2 are waves numbers in the x_1 and x_2 directions. Eq. (13) satisfies Eq. (5) when

$$\xi_2^2 = \xi_1^2 \left(\frac{v^2}{v_T^2} - 1 \right), \quad (14)$$

where the wave speed v is given by $\xi_1^2 = \omega^2 / v^2$ and $v_T^2 = \bar{c} / \rho$ is the speed of plane shear waves propagating in the x_1 direction.

Substitution of Eq. (13) into the boundary conditions at $x_2 = \pm h$ in Eq. (12) yields four linear, homogeneous algebraic equations for A_1 through A_4 . For nontrivial solutions of the undetermined constants, the determinant of the coefficient matrix of the linear equations has to vanish, which leads to the following dispersion relation of the waves

$$\frac{\tan \xi_2 h}{\tanh \xi_1 h} = \bar{k}^{-2} \frac{\xi_2}{\xi_1}, \quad (\text{AS}) \quad (15)$$

$$\frac{\tan \xi_2 h}{\tanh \xi_1 h} = -\bar{k}^2 \frac{\xi_1}{\xi_2}, \quad (\text{S}) \quad (16)$$

where AS denotes the antisymmetric mode and S the symmetric mode.

3.2 Piezoelectric plate with unattached electrodes

The wave propagation specified by Eqs. (6-8) must satisfy the following boundary and continuity conditions

$$\begin{aligned}\varphi^T &= 0, \quad x_2 = h + g^T, \\ \varphi &= \varphi^T, \quad D_2 = D_2^T, \quad T_{23} = 0, \quad x_2 = h, \\ \varphi &= \varphi^B, \quad D_2 = D_2^B, \quad T_{23} = 0, \quad x_2 = -h, \\ \varphi^B &= 0, \quad x_2 = -h - g^B.\end{aligned}\tag{17}$$

Consider the following propagating waves in the plate

$$\begin{aligned}u &= (A_1 \sin \xi_2 x_2 + A_2 \cos \xi_2 x_2) \cos(\xi_1 x_1 - \omega t), \\ \psi &= (A_3 \sinh \xi_1 x_2 + A_4 \cosh \xi_1 x_2) \cos(\xi_1 x_1 - \omega t),\end{aligned}\tag{18}$$

and the waves in the gaps

$$\varphi^T = A_5 \sinh \xi_1 (x_2 - h - g^T) \cos(\xi_1 x_1 - \omega t),\tag{19}$$

$$\varphi^B = A_6 \sinh \xi_1 (x_2 + h + g^B) \cos(\xi_1 x_1 - \omega t),\tag{20}$$

where A_1 through A_4 are undetermined constants, ω is the wave frequency, ξ_1 and ξ_2 are defined in Eq. (14). A_5 and A_6 are also undetermined constants. Eqs. (19) and (20) satisfy Eqs. (7) and (8), respectively, as well as the boundary conditions at the top and bottom electrodes, i.e., the first and last equations in Eq. (17).

Substitution of Eqs. (18), (19) and (20) into the remaining boundary and continuity conditions at $x_2 = \pm h$ in Eq. (17) yields six linear, homogeneous algebraic equations for A_1 through A_6 . For nontrivial solutions of the undetermined constants, the determinant of the coefficient matrix of the linear equations has to vanish, which leads to the following dispersion relation of the waves:

$$\begin{aligned}& \frac{n^2 \tanh \xi_1 g^T + \tanh \xi_1 h - \bar{k}^2 \frac{\xi_1}{\xi_2} \tan \xi_2 h}{n^2 \tanh \xi_1 g^B + \tanh \xi_1 h - \bar{k}^2 \frac{\xi_1}{\xi_2} \tan \xi_2 h} \\ &= - \frac{n^2 \tanh \xi_1 g^T + \text{ctanh} \xi_1 h + \bar{k}^2 \frac{\xi_1}{\xi_2} \text{ctan} \xi_2 h}{n^2 \tanh \xi_1 g^B + \text{ctanh} \xi_1 h + \bar{k}^2 \frac{\xi_1}{\xi_2} \text{ctan} \xi_2 h},\end{aligned}\tag{21}$$

where $n^2 = \varepsilon / \varepsilon_0$ is the refractive index in the x_1 direction, ε_0 is the permittivity of free space, and $\bar{k}^2 = e^2 / (\varepsilon \bar{c})$ is a dimensionless number (electromechanical coupling factor). With Eq. (14), Eq. (21) determines v versus ξ_1 or ω versus ξ_1 .

We examine some special cases below. When the gaps have the same thickness, i.e., $g^T = g^B = g$, Eq. (11) factors into two equations that determine two groups of waves. One may be called anti-symmetric waves and the other symmetric. The displacement u of the anti-symmetric and symmetric waves are odd and even functions of x_2 , respectively. The corresponding dispersion relations are

$$n^2 \tanh \xi_1 g + \tanh \xi_1 h - \bar{k}^2 \frac{\xi_1}{\xi_2} \tan \xi_2 h = 0, \quad (\text{AS})\tag{22}$$

$$n^2 \tanh \xi_1 g + \operatorname{ctanh} \xi_1 h + \bar{k}^2 \frac{\xi_1}{\xi_2} \operatorname{ctan} \xi_2 h = 0. \quad (\text{S}) \quad (23)$$

In particular, if the gaps are not present, i.e., when $g^T = g^B = 0$, Eqs. (22) and (23) reduce exactly to Eqs. (15) and (16).

3.3 Piezoelectric plate in contact with viscous fluids

The wave propagation specified by Eqs. (9-11) must satisfy the following boundary and continuity conditions

$$\begin{aligned} \varphi^T &= 0, \quad v^T = 0 \quad \text{at} \quad x_2 = h + g^T, \\ \varphi &= \varphi^T, \quad D_2 = D_2^T, \quad T_{23} = T_{23}^T, \quad v = v^T \quad \text{at} \quad x_2 = h, \\ \varphi &= \varphi^B, \quad D_2 = D_2^B, \quad T_{23} = T_{23}^B, \quad v = v^B \quad \text{at} \quad x_2 = -h, \\ \varphi^B &= 0, \quad v^B = 0 \quad \text{at} \quad x_2 = -h - g^B. \end{aligned} \quad (24)$$

We look for solutions representing waves propagating in the x_1 direction in the following form

$$\begin{aligned} u_3 &= (A_1 \sin \xi_2 x_2 + A_2 \cos \xi_2 x_2) \exp i(\xi_1 x_1 - \omega t) \\ \psi &= (A_3 \sinh \xi_1 x_2 + A_4 \cosh \xi_1 x_2) \exp i(\xi_1 x_1 - \omega t) \end{aligned} \quad -h < x_2 < h, \quad (25)$$

$$\begin{aligned} v^T &= A_7 \sinh \xi_2^T (x_2 - h - g^T) \exp i(\xi_1 x_1 - \omega t) \\ \varphi^T &= A_5 \sinh \xi_1 (x_2 - h - g^T) \exp i(\xi_1 x_1 - \omega t) \end{aligned} \quad h \leq x_2 \leq h + g^T, \quad (26)$$

$$\begin{aligned} v^B &= A_8 \sinh \xi_2^B (x_2 + h + g^B) \exp i(\xi_1 x_1 - \omega t) \\ \varphi^B &= A_6 \sinh \xi_1 (x_2 + h + g^B) \exp i(\xi_1 x_1 - \omega t) \end{aligned} \quad -h - g^B \leq x_2 \leq -h, \quad (27)$$

where ω is the wave frequency, and ξ_1 and ξ_2 are the wave numbers in the x_1 and x_2 directions, as defined in Eq. (14). A_1 through A_8 are undetermined constants. (25)-(27) already satisfy the boundary conditions in (24) at $x_2 = h + g^T$ and $x_2 = -h - g^B$. (25)-(27) also satisfy (9)-(11) provided that

$$\begin{aligned} (\xi_2^T)^2 &= \xi_1^2 - i\omega \frac{\rho^T}{\mu^T} = \xi_1^2 \left(1 - i \frac{\nu}{\xi_1} \frac{\rho^T}{\mu^T} \right), \\ (\xi_2^B)^2 &= \xi_1^2 - i\omega \frac{\rho^B}{\mu^B} = \xi_1^2 \left(1 - i \frac{\nu}{\xi_1} \frac{\rho^B}{\mu^B} \right), \end{aligned} \quad (28)$$

Substitution of (25)-(27) and their stress and electronic displacement components into the eight continuity conditions in (24) at $x_2 = \pm h$ results in the following eight linear, homogeneous equations for A_1 through A_8 :

$$\begin{aligned} A_5 \sinh(-\xi_1 g^T) &= A_3 \sinh \xi_1 h + A_4 \cosh \xi_1 h + \frac{\varepsilon}{\varepsilon} (A_1 \sin \xi_2 h + A_2 \cos \xi_2 h), \\ -\varepsilon^T A_5 \xi_1 \cosh \xi_1 g^T &= -\varepsilon (A_3 \xi_1 \cosh \xi_1 h + A_4 \xi_1 \sinh \xi_1 h), \end{aligned} \quad (29a)$$

$$\begin{aligned} & \bar{c} (A_1 \xi_2 \cos \xi_2 h - A_2 \xi_2 \sin \xi_2 h) + e (A_3 \xi_1 \cosh \xi_1 h + A_4 \xi_1 \sinh \xi_1 h) \\ & = \mu^T \xi_2^T A_7 \cosh \xi_2^T g^T, \end{aligned} \quad (29b)$$

$$-(A_1 \sin \xi_2 h + A_2 \cos \xi_2 h) i \omega = A_6 \sinh \xi_1 g^B = \quad (29c)$$

$$(A_1 \sin \xi_2 h - A_2 \cos \xi_2 h) i \omega = A_8 \sinh \xi_2^B g^B, \quad (29d)$$

$$\begin{aligned} & \bar{c} (A_1 \xi_2 \cos \xi_2 h + A_2 \xi_2 \sin \xi_2 h) + e (A_3 \xi_1 \cosh \xi_1 h - A_4 \xi_1 \sinh \xi_1 h) \\ & = \mu^B \xi_2^B A_8 \cosh \xi_2^B g^B, \end{aligned} \quad (29e)$$

$$-\varepsilon^B A_6 \xi_1 \cosh \xi_1 g^B = -\varepsilon (A_3 \xi_1 \cosh \xi_1 h - A_4 \xi_1 \sinh \xi_1 h), \quad (29f)$$

$$-A_3 \sinh \xi_1 h + A_4 \cosh \xi_1 h + \frac{\varepsilon}{\varepsilon} (-A_1 \sin \xi_2 h + A_2 \cos \xi_2 h). \quad (29g)$$

Eq. (29) can be written as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} A_3 \\ A_4 \end{bmatrix} = 0, \quad (30)$$

where

$$\begin{aligned} M_{11} = & -\frac{\frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T + \frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B}{2 \tan \xi_2 h} - \frac{\tanh \xi_1 h}{\tan \xi_2 h} - \frac{\tan \xi_2 h}{2} \left(\frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B - \frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T \right) \\ & + \frac{e^2}{\varepsilon \bar{c}} \frac{\xi_1}{\xi_2} + \frac{i \omega \mu^T \xi_2^T}{\bar{c} \xi_2 \tanh \xi_2^T g^T} \left(\frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T + \tanh \xi_1 h \right), \end{aligned} \quad (31)$$

$$\begin{aligned} M_{12} = & \frac{\tanh \xi_1 h}{2 \tan \xi_2 h} \left(\frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B - \frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T \right) + \tan \xi_2 h + \frac{\tanh \xi_1 h}{2 \tan \xi_2 h} \left(\frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T + \frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B \right) \\ & + \frac{e^2}{\varepsilon \bar{c}} \frac{\xi_1}{\xi_2} \tanh \xi_1 h + \frac{i \omega \mu^T \xi_2^T}{\bar{c} \xi_2 \tanh \xi_2^T g^T} \left(\frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T \tanh \xi_1 h + 1 \right), \end{aligned} \quad (32)$$

$$\begin{aligned} M_{21} = & -\frac{\frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T + \frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B}{2 \tan \xi_2 h} - \frac{\tanh \xi_1 h}{\tan \xi_2 h} + \frac{\tan \xi_2 h}{2} \left(\frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B - \frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T \right) \\ & + \frac{e^2}{\varepsilon \bar{c}} \frac{\xi_1}{\xi_2} + \frac{i \omega \mu^B \xi_2^B}{\bar{c} \xi_2 \tanh \xi_2^B g^B} \left(\frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B + \tanh \xi_1 h \right), \end{aligned} \quad (33)$$

$$\begin{aligned} M_{22} = & \frac{\tanh \xi_1 h}{2 \tan \xi_2 h} \left(\frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B - \frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T \right) - \tan \xi_2 h - \frac{\tanh \xi_1 h}{2 \tan \xi_2 h} \left(\frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T + \frac{\varepsilon}{\varepsilon_B} \tanh \xi_1 g^B \right) \\ & - \frac{e^2}{\varepsilon \bar{c}} \frac{\xi_1}{\xi_2} \tanh \xi_1 h - \frac{i \omega \mu^B \xi_2^B}{\bar{c} \xi_2 \tanh \xi_2^B g^B} \left(\frac{\varepsilon}{\varepsilon_T} \tanh \xi_1 g^T \tanh \xi_1 h + 1 \right). \end{aligned} \quad (34)$$

The dispersion relation of the waves, i.e., ω versus ξ_1 , is determined by the following equation:

$$M_{11}M_{22} - M_{12}M_{21} = 0, \quad (35)$$

which is equivalent to that the determinant of the coefficient matrix of (29) vanishes.

Expression (35) is a long equation when expanded. Below we consider the special case when the structure is symmetric about $x_2 = 0$, i.e.,

$$g^T = g^B = g, \quad \mu^T = \mu^B = \mu, \quad \rho^T = \rho^B = \rho^L, \quad \varepsilon^T = \varepsilon^B = \varepsilon. \quad (36)$$

In this case, $\xi_2^T = \xi_2^B = \xi$. The waves separate into antisymmetric and symmetric ones about $x_2 = 0$. They are odd or even functions of x_2 , respectively. (35) factors into two equations determining the dispersion relations of the antisymmetric (AS) and symmetric (S) waves:

$$\frac{\tanh \xi_1 h}{\tanh \xi_2 h} + \frac{\varepsilon_{11}}{\varepsilon} \frac{\tanh \xi_1 g}{\tanh \xi_2 h} - \frac{e_{15}^2}{\varepsilon_{11} \bar{c}_{44}} \frac{\xi_1}{\xi_2} - \frac{i\omega\mu\xi}{\bar{c}_{44}\xi_2 \tanh \xi g} \left(\frac{\varepsilon_{11}}{\varepsilon} \tanh \xi_1 g + \tanh \xi_1 h \right) = 0, \quad (\text{AS}) \quad (37)$$

$$\frac{\tanh \xi_2 h}{\tanh \xi_1 h} + \frac{\varepsilon_{11}}{\varepsilon} \frac{\tanh \xi_1 g}{\tanh \xi_2 h} + \frac{e_{15}^2}{\varepsilon_{11} \bar{c}_{44}} \frac{\xi_1}{\xi_2} + \frac{i\omega\mu\xi}{\bar{c}_{44}\xi_2 \tanh \xi g} \left(\frac{\varepsilon_{11}}{\varepsilon} \tanh \xi_1 g + \tanh \xi_1 h \right) = 0. \quad (\text{S}) \quad (38)$$

(37) or (38) can be viewed as an equation for v (or ω) with the use of (28). We make the following observations from (37) or (38):

- i. The waves are dispersive due to the presence of ξ_1 in (37) or (38), in addition to v (or ω). Waves in plates without contacting a fluid are already dispersive in general (Bleustein, 1969). Fluids cause additional dispersion.
- ii. Due to the presence of the fluid, ξ_1 is complex as suggested by (28). Then the dispersion relations determined by (37) or (38) are complex in general. These are damped waves or waves with attenuation as expected. This is fundamentally different from the case without fluids (Bleustein, 1969) where many real branches of dispersion relations can be obtained. If we keep ξ_1 real, (37) or (38) determines a complex v (or ω). If we keep v (or ω) real, (37) or (38) determines a complex ξ_1 . In this case the phase velocity is calculated by $v = \omega / \text{Re}(\xi_1)$, and the imaginary part of ξ_1 describes attenuation in the propagation direction.
- iii. If the fluids are not present, we have air gaps. we can reduce (37) and (38) exactly into Eqs. (22) and (23).
- iv. If we remove the gaps, i.e., $g^T = g^B = g = 0$, then the reduced results in (iii) further reduce to Eqs. (15) and (16) exactly.

4. Numerical results

4.1 Dispersion curves for thickness-twist waves in an infinite piezoelectric plate

Following Bleustein (1969), we introduce the following dimensionless frequency Ω and the dimensionless wave number Z by:

$$\Omega^2 = \omega^2 / \left(\frac{\pi^2 \bar{c}}{4\rho h^2} \right), \quad Z = \xi_1 / \left(\frac{\pi}{2h} \right). \quad (39)$$

Then the dispersion relations (15) and (16) can be rewritten as

$$\frac{\tan(\pi/2)(\Omega^2 - Z^2)^{1/2}}{\tanh(\pi/2)Z} = \frac{(\Omega^2 - Z^2)^{1/2}}{\bar{k}^2 Z} \quad (\text{AS}) \quad (40)$$

$$\frac{\tan(\pi/2)(\Omega^2 - Z^2)^{1/2}}{\tanh(\pi/2)Z} = \frac{-\bar{k}^2 Z}{(\Omega^2 - Z^2)^{1/2}} \quad (\text{S}) \quad (41)$$

The roots of Eqs. (40) and (41) determine the dispersion relation for the thickness-twist waves propagating in the x_1 direction. We note that the nondimensional frequency, Ω , must be real and positive, however the nondimensional wave number, Z , may be real, imaginary, or complex. It is clear from Eqs. (40) and (41) that if Z is a root (for a given Ω), so also is $-Z$. Segments of the first few positive real and imaginary branches of Eqs. (40) and (41) (computed for PZT-5H) are plotted as dashed lines and solid lines in Fig. 4, respectively.

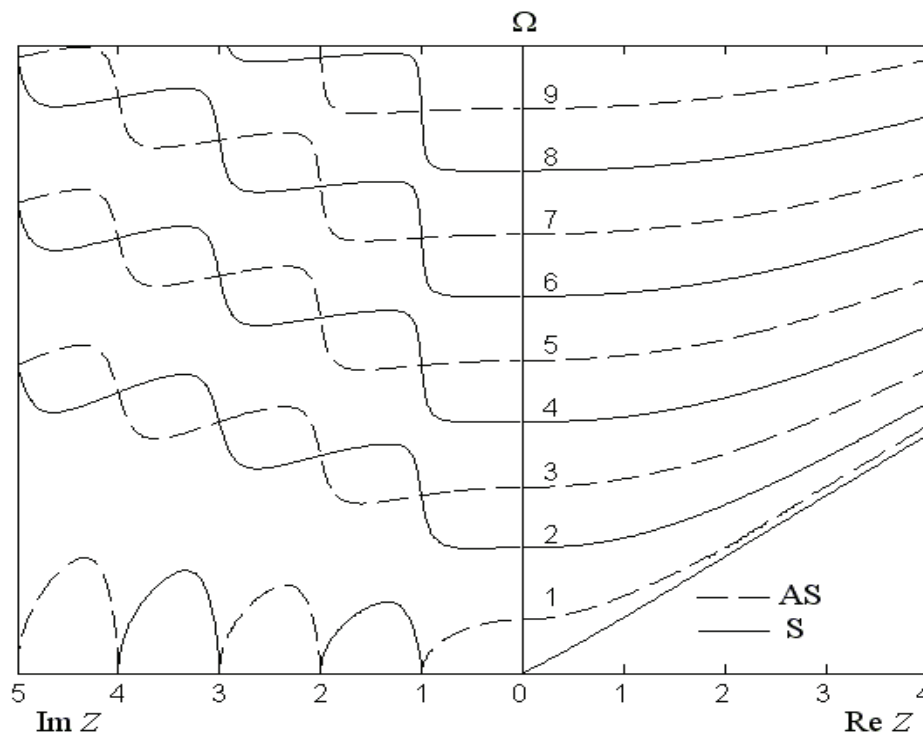


Fig. 4. Antisymmetric and symmetric real and pure imaginary branches

4.2 Effect of air gaps on dispersion curves

We here introduce a thickness ratio $m=g/h$. Numerical results for Ω versus Z are obtained by solving Eqs. (22) and (23) using PZT-5H, and are shown in Figs. 5 and 6, respectively, for $m=0$ and $m=0.0001$. The case of $m=0$ is determined by Eqs. (15) and (16) and was first given in Bleustein (1969). The figures show that the frequencies of short waves with a large wave number Z are more sensitive to m than long waves with a small Z . For a very small $m=0.0001$, the effect on short waves is already significant. The gaps raise the frequencies of short waves with a large Z .

To examine the effect of the gaps on long waves with a small Z , we plot the case of $m=0.1$ in Fig. 7 for anti-symmetric and symmetric waves together. The anti-symmetric waves are more sensitive to the air gaps, especially the lower-order modes. The symmetric waves are almost unaffected. The seemingly vertical line when Z is imaginary is in fact due to several

branches of the dispersion relation. Those almost straight lines parallel to the vertical axis correspond to the non-horizontal solid lines closest to the vertical axis in the imaginary part

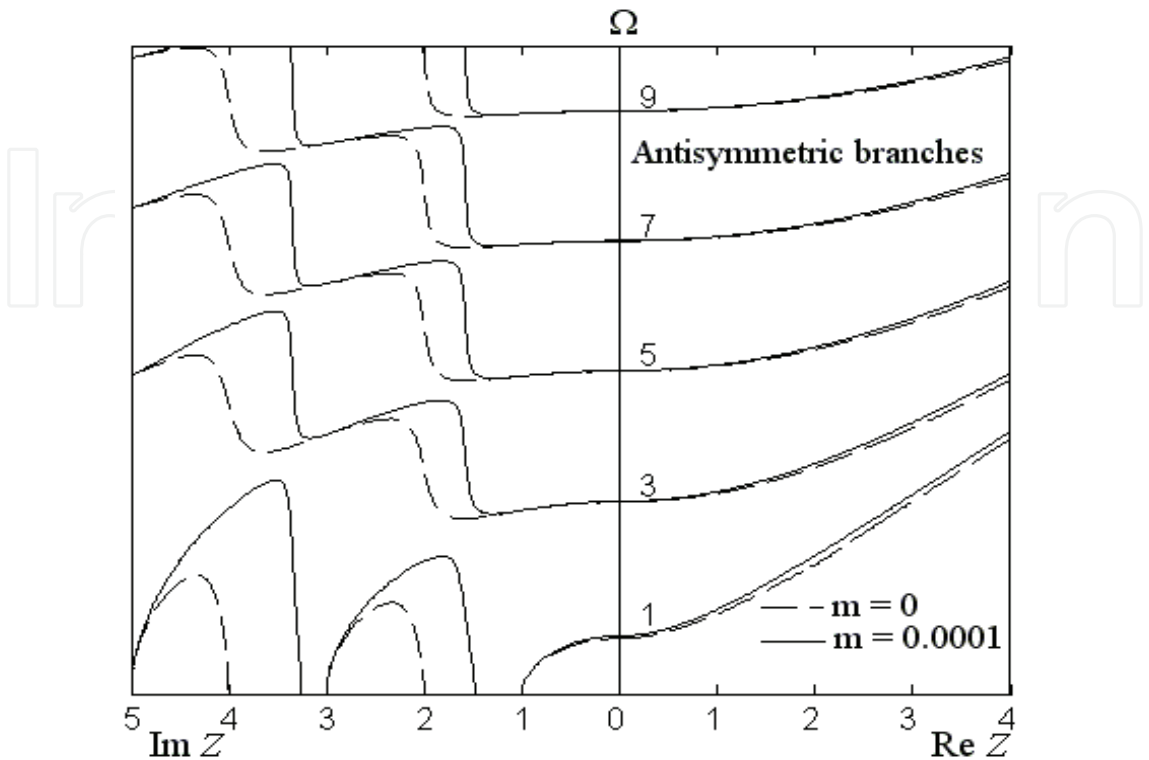


Fig. 5. Effect of air gaps on short anti-symmetric waves

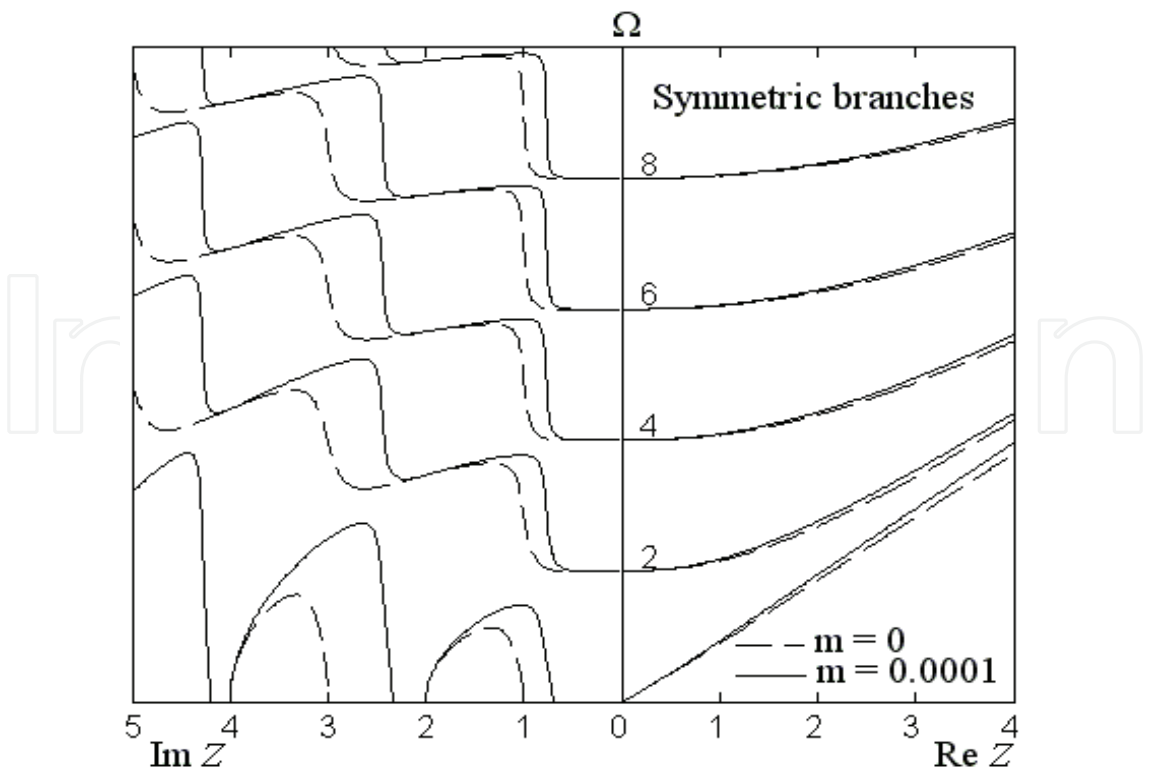


Fig. 6. Effect of air gaps on short symmetric waves

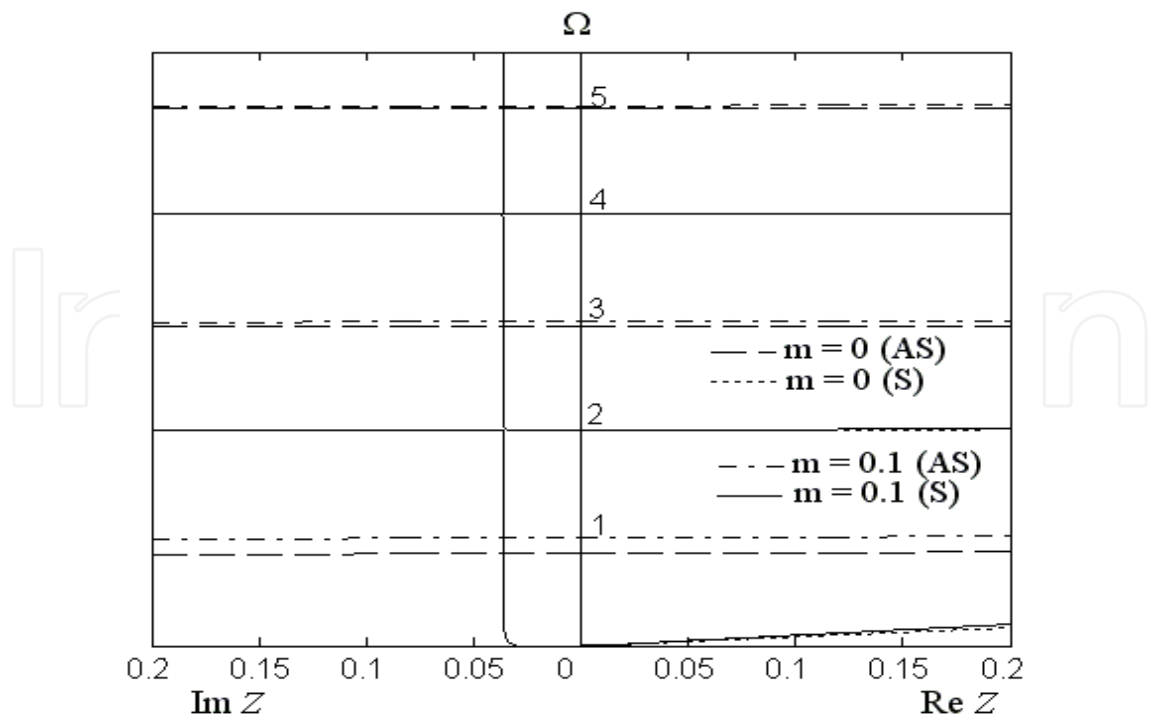


Fig. 7. Effect of air gaps on long waves

of Fig. 6. Again the gaps raise the frequencies. In Yang et al. (2009) it was found that for pure thickness-shear waves with $Z=0$ in quartz plates the gaps also raise the frequencies. This seems to be a common effect and can be explained as follows. When the electrodes are farther apart, the shortening effect of the electrodes becomes less. Therefore stronger electric fields and the related piezoelectric stiffening effect may develop in the plate and hence the frequencies become higher.

The dependence on and sensitivity to m can be used in the design of acoustic wave devices to adjust the behavior of the waves. For example, gaps of varying thickness can be used to realize energy trapping of thickness waves in a plate (Yang et al., 2009), a phenomenon in which the waves are confined in a portion of the plate. Energy trapping is crucial to device mounting in applications.

4.3 Viscosity-induced frequency shift

To see the effect of the fluid viscosity on the wave propagation characteristics in further detail, we examine a specific case. Consider the antisymmetric waves described by Eq. (37) when $g = \infty$. When the fluid has a low viscosity, i.e., μ is small, we use an iteration (or perturbation) procedure. As the lowest order of approximation, we neglect the effect of the fluid viscosity by setting $\mu = 0$ and in Eq. (37) and obtain

$$\frac{\tanh \xi_1 h}{\tan \xi_2^{(0)} h} + \frac{\varepsilon}{\varepsilon} \frac{1}{\tan \xi_2^{(0)} h} - \frac{e^2}{\varepsilon \bar{c}} \frac{\xi_1}{\xi_2^{(0)}} = 0, \tag{42}$$

where, for a given ξ_1 , we have denoted the wave frequency when the viscosity is neglected by $\omega^{(0)}$ and

$$\left(\xi_2^{(0)}\right)^2 = \left(\omega^{(0)}\right)^2 / v_T^2 - \xi_1^2. \tag{43}$$

Given a wave number ξ_1 , Eq. (42) determines a series of frequencies $\omega^{(0)}(\xi_1)$ for guided waves in the plate. When the fluid viscosity is considered, these wave frequencies are perturbed and are determined by Eq. (37). Let the corresponding frequencies from Eq. (37) be denoted by

$$\omega = \omega^{(0)} + \Delta\omega. \quad (44)$$

Substituting Eq. (44) into Eq. (37), we obtain the following first-order modification of the wave frequencies due to the fluid viscosity:

$$\frac{\Delta\omega}{\omega^{(0)}} \cong \frac{v_T^2 \xi_2^{(0)}}{\omega^{(0)}} \frac{i\mu \xi^{(0)}}{\bar{c}} \frac{\tan(\xi_2^{(0)}h) \left(\frac{\varepsilon}{\varepsilon} + \tanh(\xi_1 h) \right)}{\frac{\varepsilon}{\varepsilon} + \tanh(\xi_1 h) - \frac{e^2}{\varepsilon \bar{c}} \frac{\xi_1 h}{\cos^2(\xi_2^{(0)}h)}}, \quad (45)$$

where

$$(\xi^{(0)})^2 = \xi_1^2 - i\omega^{(0)} \frac{\rho^L}{\mu}. \quad (46)$$

Equation (45) determines the frequency perturbation due to the fluid viscosity. It is a complex number in general. In addition to its real part representing frequency shifts, its imaginary part describes the damping effect of the viscosity. The viscosity-induced frequency shift can be used for viscosity measurement.

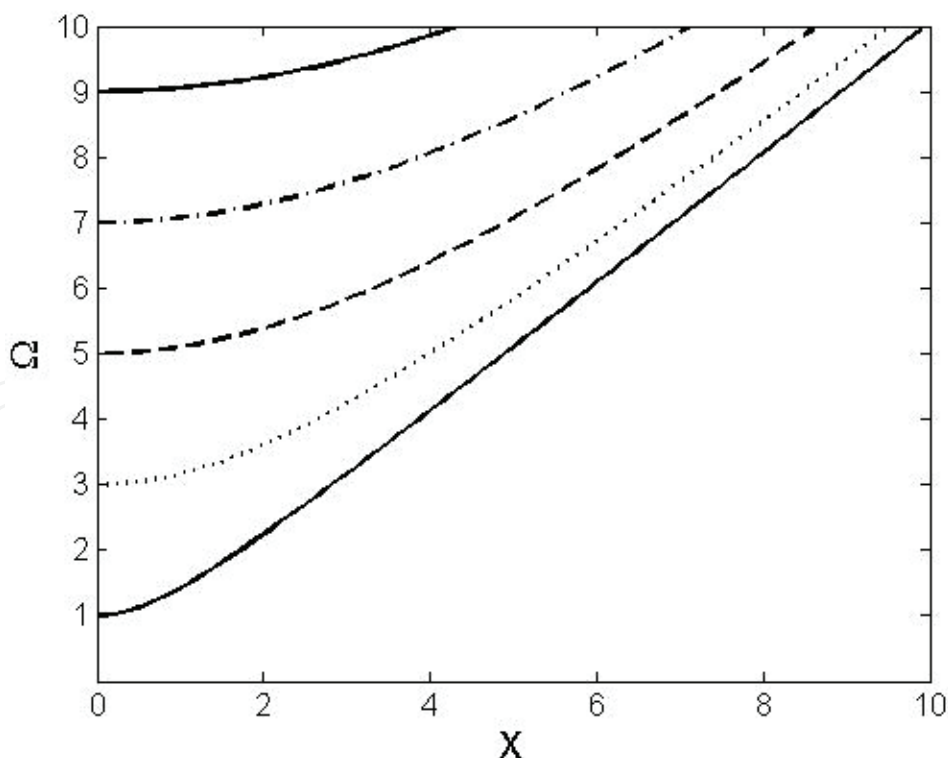


Fig. 8. Dispersion curves of Eq. (37) when fluid viscosity is neglected

For numerical results we consider a ceramic plate of PZT-5H in water, a low viscosity fluid. Using the dimensionless wave number X and the dimensionless frequency Ω defined in Eq.

(39), we plot the dispersion curves determined by Eq. (42) in Fig. 8. These are odd thickness-twist waves. Only the first five branches are shown. In applications long waves ($X \ll 1$) of the lowest branch (the fundamental mode) are used most often. Therefore in Fig. 9 we present the frequency shift calculated from the real part of Eq. (45) for long, fundamental thickness-twist waves. In acoustic wave sensors, thermal noise is usually of the order of 10^{-6} in terms of the relative frequency shift. Therefore the frequency shift in Fig. 9 is considered a clear signal.

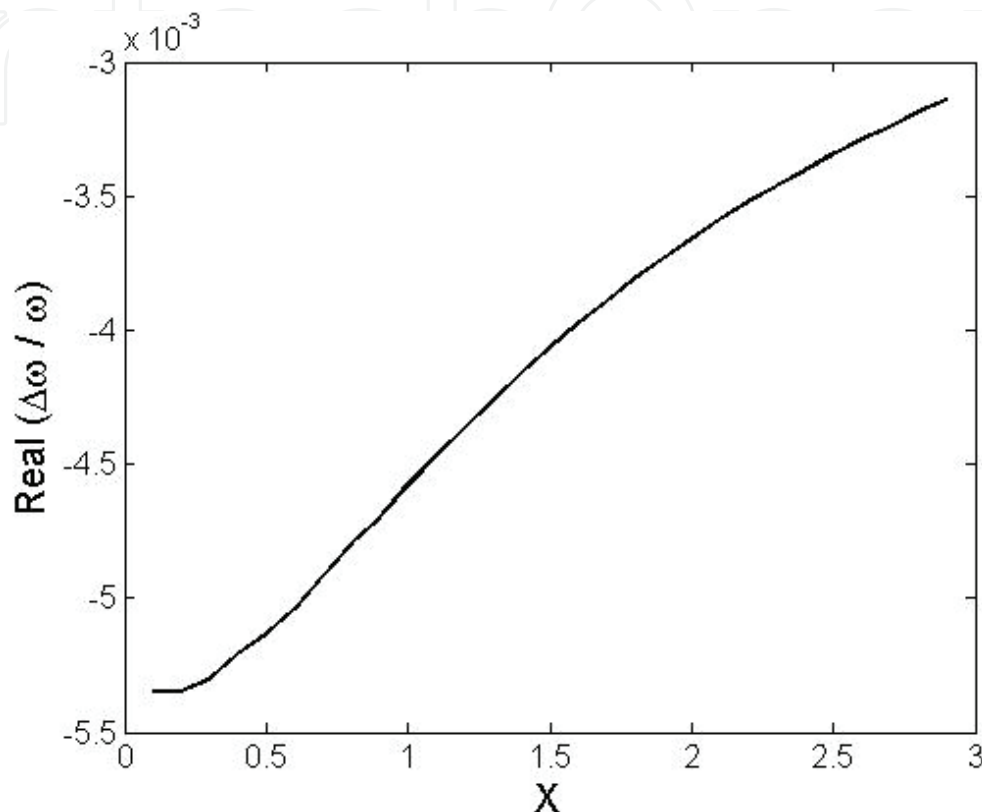


Fig. 9. Frequency shift of the lowest thickness-twist wave due to fluid viscosity

5. Conclusion

In this Chapter, we study the propagation of thickness-twist waves in an infinite piezoelectric ceramic plate. Three cases are taken into account: 1) Propagation of thickness-twist waves in a piezoelectric ceramic plate with attached electrodes; 2) Propagation of thickness-twist waves in a piezoelectric ceramic plate with unattached electrodes; 3) Propagation of thickness-twist waves in a piezoelectric ceramic plate in contact with viscous fluids. The first part is used as a basic introduction for those who want to conduct the research work on thickness-twist waves in an infinite piezoelectric ceramic plate. We mainly focus on the latter two parts. A simple solution for thickness-twist waves propagating in ceramics plates with unattached electrodes and air gaps is obtained. In the case of symmetric gaps the waves separate into symmetric and antisymmetric ones. The wave frequency is sensitive to the gap thickness, especially for short waves. In the examples calculated the gaps raise the wave frequencies. The dependence of frequency on air gaps provides a new design factor for acoustic wave devices. Equations determining the

dispersion relations for thickness-twist waves in a piezoelectric ceramic plate with unattached electrodes and viscous fluids under the electrodes are obtained. The dispersion relations are complex in general, representing damped waves. The equations reduce to the case of a plate with unattached electrodes and empty air gaps (Qian et al., 2009) and the case of a plate with attached electrodes (Bleustein, 1969) as special cases. The viscosity-induced frequency shift is studied in detail by the the lowest order of perturbation approximation . The results are useful for developing plate fluid sensors.

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In the recent decades, there has been a growing interest in micro- and nanotechnology. The advances in nanotechnology give rise to new applications and new types of materials with unique electromagnetic and mechanical properties. This book is devoted to the modern methods in electrodynamics and acoustics, which have been developed to describe wave propagation in these modern materials and nanodevices. The book consists of original works of leading scientists in the field of wave propagation who produced new theoretical and experimental methods in the research field and obtained new and important results. The first part of the book consists of chapters with general mathematical methods and approaches to the problem of wave propagation. A special attention is attracted to the advanced numerical methods fruitfully applied in the field of wave propagation. The second part of the book is devoted to the problems of wave propagation in newly developed metamaterials, micro- and nanostructures and porous media. In this part the interested reader will find important and fundamental results on electromagnetic wave propagation in media with negative refraction index and electromagnetic imaging in devices based on the materials. The third part of the book is devoted to the problems of wave propagation in elastic and piezoelectric media. In the fourth part, the works on the problems of wave propagation in plasma are collected. The fifth, sixth and seventh parts are devoted to the problems of wave propagation in media with chemical reactions, in nonlinear and disperse media, respectively. And finally, in the eighth part of the book some experimental methods in wave propagations are considered. It is necessary to emphasize that this book is not a textbook. It is important that the results combined in it are taken “from the desks of researchers”. Therefore, I am sure that in this book the interested and actively working readers (scientists, engineers and students) will find many interesting results and new ideas.

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